Constraint Programming Letters 1 (2006) 1-48 Submitted 6/2006; Published 7/2006

A Filtering Algorithm for Constraints of Difference in CSPs

Jean-Charles Régin JCREGIN@ILOG.SA
Ilog Sophia Antipolis
Les Taissounières HB2
1681, route des Dolines
06560 Valbonne, FranceIlog, SA

Editor: Pascal Van Hentenryck

Abstract

Many real-life Constraint Satisfaction Problems (CSPs) involve some constraints similar to the alldifferent constraints. These constraints are called constraints of difference. They are defined on a subset of variables by a set of tuples for which the values occuring in the same tuple are all different. In this paper, a new filtering algorithm for these constraints is presented. It achieves the generalized arc-consistency condition for these non-binary constraints. It is based on matching theory and its complexity is low. In fact, for a constraint defined on a subset of p variables having domains of cardinality at most d, its space complexity is $O(pd)$ and its time complexity is $O(p^2d^2)$. This filtering algorithm has been successfully used in the system RESYN, to solve the subgraph isomorphism problem.

Keywords: Alldifferent, Filtering Algorithms, GAC-Schema

1. Introduction

The constraint satisfaction problems (CSPs) form a simple formal frame to represent and solve some problems in artificial intelligence. The problem of the existence of solutions in a CSP is NP-complete. Therefore, some methods have been developed to simplify the CSP before or during the search for solutions. The consistency techniques are the most frequently used. Several algorithms achieving arc-consistency have been proposed for binary CSPs Mackworth (1977); Mohr and Henderson (1986); Bessiere and Cordier (1993); Bessière (1994) and for n-ary CSPs Mohr (1988). Only limited works have been carried out on the semantics of contraints : Mohr and Masini (1988) have described an improvement of the algorithm AC-4 for special constraints introduced by a vision problem, Van Hentenryck et al. (1992) have studied monotonic and functional binary constraints. In this work, we are interested in a special case of n-ary constraints : the constraints of difference, for which we propose a filtering algorithm.

A constraint is called *constraint of difference* if it is defined on a subset of variables by a set of tuples for which the values occuring in the same tuple are all different. They are present in many real-life problems.

These constraints can be represented as n-ary constraints and filtered by the generalized arc-consistency algorithm GAC4 Mohr (1988). This filtering efficiently reduces the domains but its complexity can be expensive. In fact, it depends on the length and the number of all admissible tuples. Let us consider a constraint of difference defined on p variables, which take their values in a set of cardinality d. Thus, the number of admissible tuples corresponds to the number of permutations of p elements selected from d elements without repetition : ${}^{d}P_{p} = \frac{d!}{(d-p)!}$. Therefore some constraint resolution systems, like CHIP Van Hentenryck (1989), represent these n-ary constraints by sets of binary constraints. In this case, a binary constraint of difference is built for each pair of variables belonging to the same constraint of difference. But the pruning performance of arc-consistency, for these constraints is poor. In fact, for a binary alldifferent constraint between two variables i and j, arc-consistency removes a value from domain of i only when the domain of j is reduced to a single value.

Acknowledgments

We would like to thank particularly Christian Bessière and also Marie-Catherine Vilarem, Tibor Kokény and the anonymous reviewers for their comments which helped improve this paper.

References

C. Bessière. Arc-consistency and arc-consistency again. *Artificial Intelligence*, 65(1):179–190, 1994.

C. Bessiere and M.-O. Cordier. Arc-consistency and arc-consitency again. In *Proceedings of the Eleventh National Conference on Artificial Intelligence (AAAI-93)*, pages 108–113, Washington, DC, 1993.

A.K. Mackworth. Consistency in Networks of Relations. *Artificial Intelligence*, 8(1):99–118, 1977.

R. Mohr. Good Old Discrete Relaxation. In *European Conference on Artificial Intelligence (ECAI-88)*, Munich, W. Germany, August 1988.

R. Mohr and T.C. Henderson. Arc and Path Consistency Revisited. *Artificial Intelligence*, 28:225–233, 1986.

R. Mohr and G. Masini. Running Efficiently Arc Consistency. In *Syntactic and Structural Pattern Recognition*, pages 217–231. Springer-Verlag, 1988.

P. Van Hentenryck. *Constraint Satisfaction in Logic Programming.* Logic Programming Series, The MIT Press, Cambridge, MA, 1989.

P. Van Hentenryck, Y. Deville, and C.M. Teng. A Generic Arc Consistency Algorithm and Its Specializations. *Artificial Intelligence*, 57(2-3), 1992.